Pattern selection by a granular wave in a rotating drum

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The results of an experimental investigation of granular segregation in a thin rotating drum are presented. A mechanism based on the presence of an uphill wave of particles has been found to govern the observed pattern of petals. Specifically we develop a simple model that captures the essential physics of the segregation and yields an algebraic expression that predicts the number of petals in the pattern.

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When a mix of granular material is caused to flow, segregation or demixing into the constituent components of the mixture can occur [1–4]. In the physics community this is recognized as an interesting example of spontaneous pattern formation, in geological applications it is thought to be the origin of stratification in certain rock formations [5], and in an engineering context it has undesirable consequences in the industry [1]. Segregation does not always occur and the fundamental mechanisms which give rise to it remain aloof despite decades of research [3]. One example of granular segregation which is of both practical and fundamental scientific interest is a long horizontal rotating drum. The original mixture is observed to separate into bands along the axis of the cylinder [6,7] after a period of rotation. However, segregation in the radial direction also occurs in long drums [7] such that the smaller sized particles gather on the axis of rotation to form a core. The role of the core in the axial segregation process [7,8] is not yet understood but the formation of wavelike structures along it are believed to be significant in the formation of the observed axial bands.

Radial segregation also takes place in thin rotating drums where the focus has been on the formation of a pattern of petals [9–15]. In this case axial segregation is suppressed and a direct observation of all particles is allowed [16]. As the drum rotates a thin flowing avalanche is formed at the surface. The patterns that are formed are controlled by the segregation taking place within the avalanche and the mode in which the material is deposited into the solid rotating body of grains beneath. Core segregation occurs in the continuous (rolling) regime, when small particles are trapped within the surface flow [17]. An example of a segregated core is given in Fig. 1(a). Segregation into petals is characterized by the formation of stripes similar to those developed during the formation of piles [4,5,9]. Some examples are given in Figs. 1(b)–1(d). One mechanism which has been proposed to explain the formation of stripes is wave-breaking [10]. Another model that suggests the importance of the forcing in the pattern has recently been put forward [14]. However, neither of these mechanisms appear to apply for the simplest case of a half-filled circular drum which is rotated at a constant speed. This configuration avoids geometrical effects which affect the pattern developed for different filling levels and container geometries [10,11,13,15,16]. In this paper we report the experimental results of a systematic investigation of this case and develop a model based on the presence of an uphill wave that travels from the end of the petals to the center of the drum. The existence of uphill waves has been reported before in pile formation [5,9] and chute and drum flows [9,13,18,19]. We have taken this effect into account to predict the number of petals generated for different rotation frequencies, volume fractions of small particles, and sizes of the drum.

The experimental apparatus consisted of separate 3 mm thick aluminum drums with diameters 12.0, 16.0, 20.0, 24.5, and 38.0 cm. Each had a glass front which allowed us direct observation of the particles. The drums were mounted vertically in bearings and half-filled with binary mixtures of glass particles: 0.12 ± 0.02 mm diameter white and 0.71 ± 0.10 mm diameter red beads. A motor drive was connected to the drums providing a constant rotation frequency which was measured to be within ± 0.002 rad/s, using an optical shaft

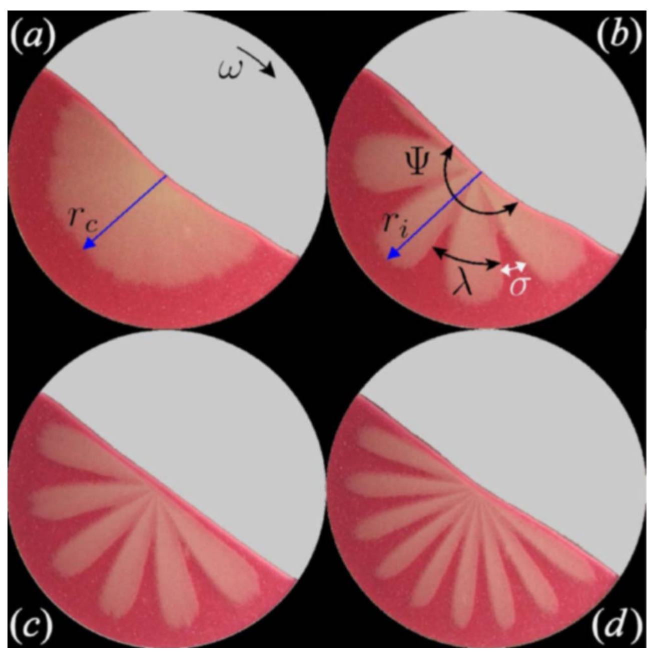

FIG. 1. (Color online) (a) core and (b)–(d) petals developed when a mixture of 0.71 mm (red/gray) and 0.12 mm (white) glass particles is rotated in a thin drum at 0.60, 0.20, 0.13, and 0.09 rad/s, respectively. The diameter of the drum is $D = 24.5$ cm and the volume fraction of small particles $\phi = 0.35$.

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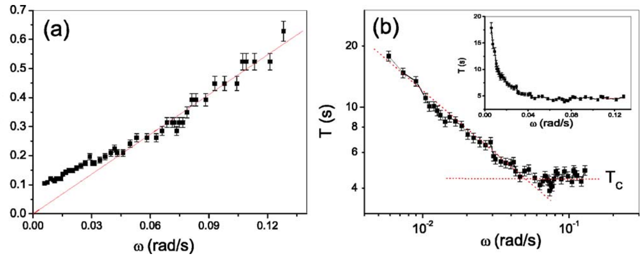

FIG. 2. (Color online) (a) Angular distance between two consecutive petals (λ) versus the rotation frequency (ω). The 24.5 cm diameter drum is half-filled with $\phi=0.42$ volume fraction of small particles. (b) The values of the period depending on the frequency of rotation that correspond to the wavelengths of the figure (a) plotted in logarithmic scale and in linear scale (inset).

encoder. Video recordings and image processing were used to study the evolution of the petals.

The experiment was performed by first rotating the drum at a high frequency (0.6 rad/s) to achieve core segregation as shown in Fig. 1(a). A nearly homogeneous semicircular distribution of small particles formed near the axis of the drum with a small amount of mixing around the edge. The frequency of the rotation was then reduced to set values and a waviness was observed to develop around the edge of the core [10]. After approximately 20 rotations of the cylinder, the pattern became fully developed. All the petals were closely matched and almost equally spaced as in the examples shown in Figs. 1(b)–1(d). The pattern can, therefore, be characterized by a wavelength λ which is defined to be the angular distance between two consecutive petals as shown in Fig. 1(b). When the frequency of rotation was close to the boundary of existence of a given number of petals, the evolution of the pattern took in excess of 2 hours before an integer number was established.

As illustrated in Figs. 1(b)–1(d) the number of petals depends on the frequency of the rotation [10,12]. When the rotation frequency (ω) was set at a prescribed value, the wavelength was found to be constant and was measured by noting the passage time T in a selected point on the pattern. Hence $\lambda=\omega T$, where λ has discrete values given by $N\lambda=\Psi$. N is the number of petals and Ψ the total angle where the petals of small particles are deposited as defined in Fig. 1(b) which is found to be slightly greater than π: $\Psi=3.3\pm0.1$ rad.

The results obtained for the dependence of the wavelength of the petals on the rotation frequency are given in Fig. 2(a). Two regions with qualitatively different behavior can be identified. For high frequencies the period is constant at $T=4.55$ s. Henceforth, T_c will be used to denote this. However, for smaller frequencies the period depends inversely on the frequency, i.e., the smaller the frequency the longer the period of the pattern. This result can be seen more clearly in Fig. 2(b), where the period is shown plotted against the frequency. For high frequencies the period is constant whereas it diverges when $\omega=0$ is approached [inset of Fig. 2(b)].

The influence of the diameter of the drum (D) and the volume fraction of small particles (ϕ) on the wavelength were investigated. The limit $\phi=1$ corresponds to pure small particles and $\phi=0$ to big particles alone. The behavior is qualitatively similar for all values of ϕ and D explored (Fig. 3). For small frequencies all the results collapse into a single curve with power law behavior: the variation in the period is independent of both ϕ and D. However, T_c does depend on both these quantities such that for a fixed frequency the larger D or ϕ the greater T_c. This yields the counterintuitive result that for a given frequency and volume fraction of small particles, the larger the drum the smaller the number of petals in the pattern.

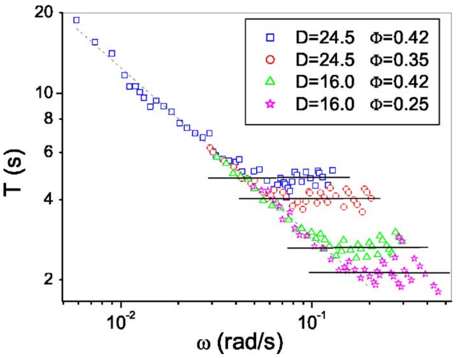

FIG. 3. (Color online) log-log plot of the period versus the frequency of the drum for different volume fractions of small particles and different diameters of the rotating drum. Solid black lines show the different T_c obtained from a fit like that in Fig. 2. The dashed black line is a least squares fit of $T=0.67w^{-0.63}$.

Since increasing D or ϕ gives rise to an increase in T_c, this suggests that the radius of the initial core [r_c in Fig. 1(a)] is the parameter that controls the selection of the period. In the ideal situation, when there is complete segregation the radius of the core can be expressed as $\sqrt{\phi}\,D/2$. However, in practice we find that the actual radius of the core is approximately $r_c=1.2\sqrt{\phi}\,D/2$ for a range of ω, D, and ϕ. We believe that the small partially mixed region adjacent to the edge of the core is the origin of this difference. In Fig. 4 we show the values of T_c versus r_c, obtained for a range of D and ϕ (from $\phi=0.1$ to $\phi=0.5$). It can be seen that T_c depends linearly on the initial radius of the core.

In order to understand the important issue of the wave-

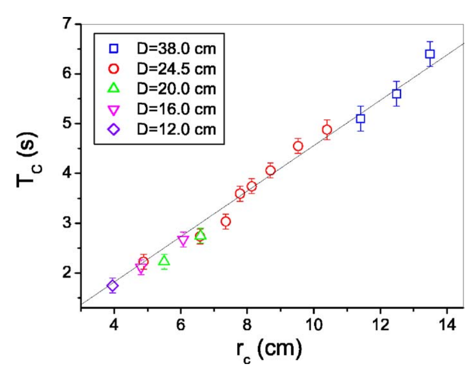

FIG. 4. (Color online) The constant period selected by the system versus the radius of the initial core of small particles. Those results have been obtained for different diameters of the drum (D) and volume fraction of small particles (ϕ). The solid black line shows a linear fit with $T_c=0.456r_c$.

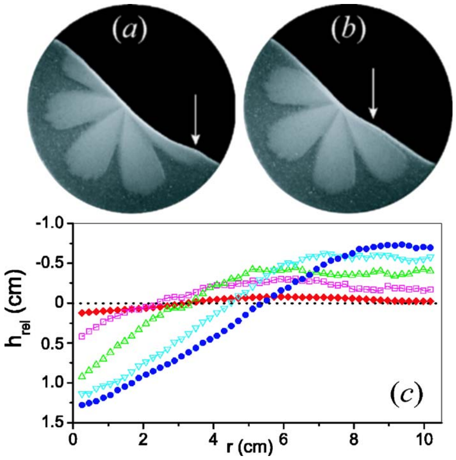

FIG. 5. (Color online) (a) Steady wave at the end of the petal during the small particles deposition. (b) Uphill wave traveling from the end of the petal to the center of the drum. (c) Surface profile after 0.0 s (•), 0.2 s (▽), 0.4 s (△), 0.6 s (□), and 0.8 s (◆) relative to the final flat surface, achieved for $t=0.88$ s (dot line). r is the distance from the center of the drum.

length selection of the pattern we now develop a model which uses the radius of the core as the controlling influence. First we need to consider the processes which take place in the avalanche. Avalanches composed of high concentrations of small particles are characterized by the presence of a standing wave located at the end of the petal indicated in Fig. 5(a).

The small particles deposit upstream of the standing wave while the large particles deposit downstream of the wave. Next, when high concentrations of large particles avalanche, the wave travels uphill from the end of the petal to the center of the drum [Fig. 5(b)]. The wave is a stopping front in the sense that in the downstream of the wave the particles remain in fixed positions in the drum frame of references [5,9,13,18,19]. The evolution of the surface profile is shown in Fig. 5(c).

Based on these observations, we propose a mechanism, that governs the pattern formation: (1) The small particles deposit and a steady wave is developed at the end of the petal. (2) When the big particles deposit the wave travels uphill from the end of the petal to the center of the drum, flattening the surface. The petals are stable if there is enough time for the uphill wave of big particles to reach the center of the drum before the next group of small particles avalanche. On the contrary, if the uphill wave has not reached the center of the drum when the small particles avalanche, they are sheared over large particles and particle-size segregation by kinetic sieving causes mixing and or separation [13]. The situation is reinforced in the following rotations and eventually causes merging of petals until they reach a stable configuration.

In our model, we consider that (in a single realization) the area occupied by the small particles is constant. This has been checked experimentally and is found to be correct within a 5% error. Then we can establish that the area of the core ($\Psi r_c^2/2$) is equal to the area of the petals which can be reasonably approximated to $\Psi r_i^2/2 - N\sigma r_i^2/2$, where N is the number of petals of length r_i and σ is the angle between two petals [Fig. 1(b)]. Then $\Psi r_c^2/2 = \Psi r_i^2/2 - N\sigma r_i^2/2$. Moreover, an approximately constant relationship has been found between the length of the petals and the initial radius of the core: $r_i = c r_c$, where $c = 1.18 \pm 0.04$. The source of this error is a very small decrease of c with ω that has been observed in experiments with small values of ϕ. Hence we will consider c to be constant. Using $\Psi r_c^2/2 = \Psi r_i^2/2 - N\sigma r_i^2/2$ we obtain

$$\Psi = \Psi c^2 - N\sigma c^2 \tag{1}$$

The minimum angle included between two petals depends on the time that it takes for the uphill wave to reach the center of the drum (τ) and ω: $\sigma = \tau\omega$. The number of petals can be related to the wavelength: $N\lambda = \Psi$. Consequently, $N\omega T_c = \Psi$. Replacing N and σ in Eq. (1)

$$1 = c^2(1 - \tau/T_c). \tag{2}$$

We can also express τ, the time that it takes for the uphill wave to reach the center of the drum, as a function of its average velocity and the distance traveled (r_i): $\tau = r_i/v$. Then by Eq. (2) and $r_i = c r_c$

$$T_c = \frac{c}{(1 - 1/c^2)v} r_c. \tag{3}$$

There are two important consequences of Eq. (3). For a fixed r_c, T_c is independent of ω provided that c and v do not depend on the frequency of rotation. This is in good agreement with observations for high frequencies as shown in Fig. 2(b). Moreover, there is a linear dependence of T_c on r_c as it is expressed by the results in Fig. 4. Hence, Eq. (3) can be used to predict the speed of the uphill wave, where $c = 1.18 \pm 0.04$ corresponds to 9.18 cm/s and the upper and lower values are 10.84 and 8.15 cm/s, respectively. The average velocity for the uphill wave can be estimated using the experimental results represented in Fig. 5 by measuring the distance traveled by the wave front for a given time interval. For different ω, D, and ϕ we measured values lying in the range between 9.2 and 11.3 cm/s which is in excellent accord with the prediction made with the model. Alternatively, assuming that the propagation speed of the uphill wave is constant, the period and hence the number of petals can be predicted with Eq. (4), where D is in cm. Clearly N is an integer number and the one that is selected is the nearest to the predicted value. At the extremities of the frequency range there is a coexistence of modes which gives rise to the long time scales discussed above.

$$N = \frac{12.06}{\omega D \sqrt{\phi}}. \tag{4}$$

The properties of the patterned segregation in a half-filled rotating drum have been characterized. The important parameters which determine the pattern are the rotation frequency, the size of the drum, and the volume fraction of small particles. The presence of an uphill wave of big particles has been observed between the avalanches of small particles. We propose that this is at the heart a new mechanism that governs the wavelength selection. Specifically, if two petals are separated by an angular distance that allows the uphill wave to reach the center of the drum, they will be stable. Otherwise mixing will occur giving rise to petal merging until a stable configuration is reached. Our model has been used to predict quantitatively the propagation speed of the wave and the wavelength of the petals, which are in accord with the observations. Clearly this important mechanism will also be relevant for patterned segregation with other fill ratios.

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